Calculation of the passive components and the commutating current in an assisted turn off inverter.

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***Abstract**: Inverters with thyristors need auxiliary turn off circuits. But even in the case of GTOs or IGBTs, an auxiliary turn off circuit has been used to reduce turn off losses. The calculation of the optimal values of the passive commutating components (C and L), and the commutating current is usually done by means of equations found by graphical or empirical methods (see ref. [1]).*

This article shows a completely analytic method, which gives more accurate results.

I. PASSIVE COMPONENTS.

Figure 1 shows a modified Mc Murray inverter (see ref. [2]). Figure 2 shows the Burgum-Nijhof inverter (see ref. [3]), which has some advantages over the previous one (less voltage over the auxiliary thyristors, no need of special sequence when starting, etc.), but as we will see later is more sensitive to the losses (low Q) in the commutating circuit. Power switches are shown as thyristors but may be GTOs or IGBTs.

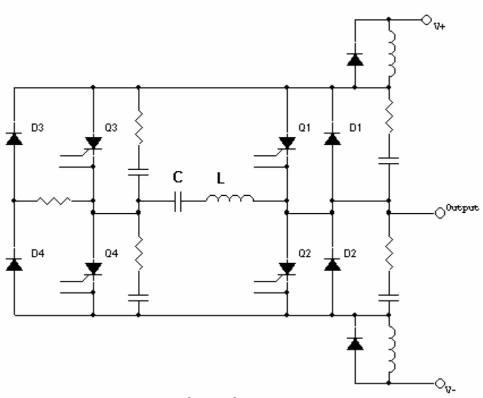

Fig. 1

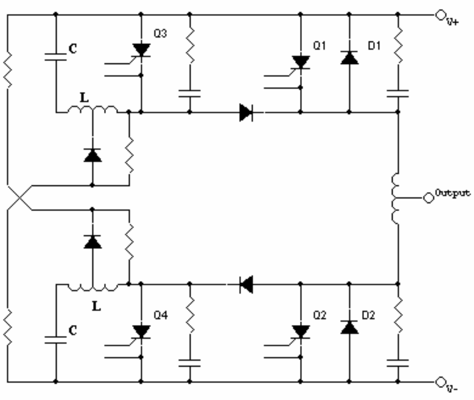

Fig. 2

Figure 3 shows the commutating current (in C and L) in the inverters of figure 1 and 2. The first inverter turns off the main thyristor in the 1st quarter of the period of the current, while the other one does it in the 3d quarter.

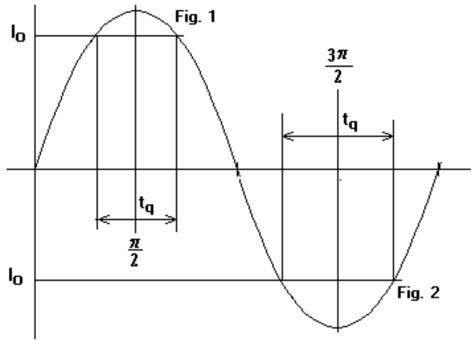

Fig. 3

We call θ the ratio between the period and the thyristor turn off time t_q:

$$\theta = \frac{T}{t_q} \qquad (1)$$

And be K the dampening factor:

$$K = e^{-\frac{\omega t}{2Q}} \qquad (2)$$

The current is a dampened sinusoid. The absolute value of the commutating current is (see fig. 3):

where n=1 for the inverter of fig. 1 and n=3 for the inverter of fig. 2.

$$I_0 = \left| K \cdot I_M \cdot \sin\left(\frac{n\pi}{2} + \frac{2\pi}{T} \cdot \frac{t_q}{2}\right)\right| = K \cdot I_M \cos\left(\frac{2\pi}{T} \cdot \frac{t_q}{2}\right)$$

$$I_M = V\sqrt{\frac{C}{L}}$$

(taking the worse case, for Q → ∞). V = minimum supply voltage - voltage drop in semiconductors. Using (1) we get:

$$I_0 = K \cdot V\sqrt{\frac{C}{L}} \cdot \cos\frac{\pi}{\theta} \qquad (3)$$

If now we use the value of L taken from $T = 2\pi\sqrt{LC}$ and the value of T given by (1) we obtain:

$$I_0 = \frac{2\pi K}{\theta \cdot t_q} V \cdot C \cdot \cos\frac{\pi}{\theta}$$ and from it we extract the value of C:

$$C = \frac{I_0 \cdot t_q}{2\pi K V} \cdot \frac{\theta}{\cos\frac{\pi}{\theta}} \qquad (4)$$

The only value we can play with is θ. We will try top find the value which gives the minimum C, thus using the minimum commutating energy 1/2 C V^2.

We take the derivative of (4) and equate it to 0:

$$\frac{\partial C}{\partial \theta} = \frac{I_0 \cdot t_q}{2\pi K V} \cdot \frac{\cos\frac{\pi}{\theta} - \theta \cdot sin\frac{\pi}{\theta} \cdot \frac{\pi}{\theta^2}}{\cos^2\frac{\pi}{\theta}} = 0$$

$$\cos\frac{\pi}{\theta} = \frac{\pi}{\theta} \cdot sin\frac{\pi}{\theta}$$

$$tan\frac{\pi}{\theta} = \frac{\theta}{\pi}$$

The solution has been solved using numerical methods (a top of end calculator, with which we have made the proof that the second derivative in this point is positive):

$\theta = 3.65159828...$ (5)

We use this value in (4) and then we divide (3) and (4) to get:

$$\boxed{C = \frac{0.8911}{K} \cdot \frac{I_0 \cdot t_q}{V}} \qquad (6)$$

$$\boxed{L = 0.379 \cdot K \cdot \frac{V \cdot t_q}{I_0}} \qquad (7)$$

For the inverter of figure 1 take ωt = π/2 for the value of K.

For the inverter of figure 2 take ωt = 3π/2.

Approximate values can be estimated taking K =1.

From (6) and (7) we get

$$\frac{t_q}{\sqrt{LC}} = \frac{2\pi}{\theta} \approx 1.7208...$$

and for the ideal case K=1, we obtain form (3) the ratio from peak to commutating current, for θ = 3.651...:

$$\frac{I_M}{I_0} = \frac{1}{\cos\frac{\pi}{\theta}} \approx 1.5333...$$

II. COMPARING THE RESULTS.

It may be interesting to compare our results with those given in ref. [1] where the value is estimated for K=1 and a graphical method:

	Our equations	Mc Murray
$t_q/\sqrt{(LC)}$	1.7208	1.68
I_M/I_0	1.533	1.5
θ	3.6516	3.74
C	0.891	0.893
L	0.379	0.397

The major discrepancy is in the value of L. Ref. [1] does not give a way to get results for values of K<1. Note that the effect of the losses is higher in the Burgum-Nijhof inverter than in the Mc Murray one, as the first commutates in the 3d quart wave of the current cycle and the second in the 1st quart wave, so the current is much less dampened. This is probably the only advantage of the Mc Murray inverter over the other one and explains probably why the author was not worried by the problem of current dampening.

Although the discrepancies between the values shown in the table above are not very important, the analytic method allows, programming the equations in a calculator or a computer, a much easy and safe way to calculate values, then compute the commutating current with available values, and calculate the effect of component tolerances.

III. CALCULATING THE COMMUTATING CURRENT FROM THE VALUES OF C AND L

The values found for L and C have to be converted to practical existing values, specially in the case of the capacitor.. Take the next value that is higher than the theoretical one. Then an estimate of the Q can be found in the prototype: it is recommended to run it at a higher voltage than the minimum, having a margin for the commutating current. Then measuring with an oscilloscope the resonant peak of the voltage, the quality factor can be estimated:

$$Q = \frac{\pi}{\ln\frac{Vs}{V_1}}$$

where Vs is the supply voltage and V_1 the peak resonant voltage.

We have achieved values of Q=40 using air cored chokes with Litz wire, and not much less using standard stranded wire. Although each individual wire is not enameled, the contact between wires occurs only in a reduced part of the surface. Solid core wire may cause Q to drop up to 20. If a ferrite core is used, care should be taken to select a low loss type for the frequency of the resonant circuit and avoid saturation.

The next step is to calculate the value of I_0 which results from the practical values of L,C and Q. We will try to do it as exact as possible, for example we estimated $I_M = V\sqrt{(C/L)}$ and this is only approximate. See a more exact equation below.

IV. Commutating current

We will follow the example of the inverter of fig. 2 and then give also the result for fig. 1.

The differential equation of the circuit is:

$$\frac{\partial^2 V}{\partial t^2}+\frac{R}{L}\cdot\frac{\partial V}{\partial t}+\frac{V}{LC}=0$$

(R being the resistance in series with L, mainly choke losses).

The general solution is

$$V(t)=e^{-\alpha t}\cdot(A\cdot\cos(\omega\cdot t)+B\cdot sin(\omega\cdot t))$$

where $\alpha=\frac{R}{2L}=\frac{\omega}{2Q}$

In our case, when t=0, V(0) = $V_M \Rightarrow$ A = $V_{M,}$ B=0 and we get:

$$V=e^{-\alpha t}\cdot V_M\cdot\cos(\omega\cdot t)$$

The current is:

$$I=-C\frac{\partial V}{\partial t}=V_M\cdot C(\omega\cdot\sin(\omega\cdot t)+\alpha\cdot\cos(\omega\cdot t))$$

$$I=V_M\sqrt{\frac{C}{L}}\cdot e^{-\frac{t}{2Q\sqrt{LC}}}\left(\sin\frac{t}{\sqrt{LC}}+\frac{1}{2Q}\cdot\cos\frac{t}{\sqrt{LC}}\right)$$

and with some manipulation we get:

(8)

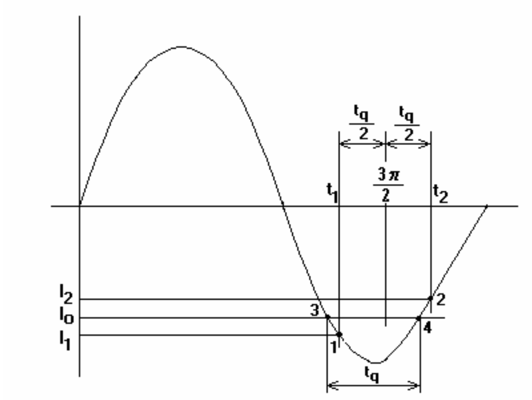

Fig. 4

As we can see in figure 4, the I_0 line does not cross the I curve in symmetrical points respect to 3/4 of the period due to the fact that the function is not a sinusoid but a dampened one. To calculate the time corresponding to the points 3 and 4 would be a terrific task as the equation is not homogeneous. We take a different approach: Be I1, I2 the currents in the symmetrical points 1 and 2. Practical cases show differences between I_1 and I_2 of 15 to 25%. If we average the values of I_1 and I_2 we should have a good approximate.

So we will use into (8) the values of t given by

$$t_1=\frac{3}{2}\pi\sqrt{LC}-\frac{t_q}{2}$$

$$t_2=\frac{3}{2}\pi\sqrt{LC}+\frac{t_q}{2}$$ and use the simplifications

sin(3π/2+β)=-cosβ, cos(3π/2+β)=-sinβ, sin(3π/2-β)=-cosβ, cos(3π/2-β)=--sinβ, to obtain:

$$I_1=V_M\sqrt{\frac{L}{C}}\cdot e^{-\frac{3\pi-\frac{t_q}{\sqrt{LC}}}{4Q}}\left(\cos\frac{t_q}{2\sqrt{LC}}+\frac{1}{2Q}sin\frac{t_q}{2\sqrt{LC}}\right)$$

$$I_2=V_M\sqrt{\frac{L}{C}}\cdot e^{-\frac{3\pi+\frac{t_q}{\sqrt{LC}}}{4Q}}\left(\cos\frac{t_q}{2\sqrt{LC}}-\frac{1}{2Q}sin\frac{t_q}{2\sqrt{LC}}\right)$$

Then we obtain the average of both values, which, after simplification gives:

$$I_0\approx\frac{I_1+I_2}{2}=\frac{1}{2}V_M\sqrt{\frac{C}{L}}\cdot e^{-\frac{3\pi}{4Q}}\left[\left(\frac{t_q}{e^{4Q\sqrt{LC}}}+\frac{-t_q}{e^{4Q\sqrt{LC}}}\right)\cdot\cos\frac{t_q}{2\sqrt{LC}}+\left(\frac{t_q}{e^{4Q\sqrt{LC}}}-\frac{-t_q}{e^{4Q\sqrt{LC}}}\right)\cdot\sin\frac{t_q}{2\sqrt{LC}}\right]$$

Taking into account that $1/2(e^{\beta}+e^{-\beta})$ = coshβ and $/2(e^{\beta}-e^{-\beta})$ = sinhβ:

$$I_0=V_M\sqrt{\frac{C}{L}}\cdot e^{-\frac{n\pi}{4Q}}\left(\cos\frac{t_q}{2\sqrt{LC}}\cdot\cosh\frac{t_q}{4Q\sqrt{LC}}+\frac{1}{2Q}\cdot\sin\frac{t_q}{2\sqrt{LC}}\cdot\sinh\frac{t_q}{4Q\sqrt{LC}}\right)\qquad(9)$$

Where n = 1 for the inverter of fig. 1 and n=3 for the inverter of fig. 2. V_M is the supply voltage less voltage drop in semiconductors.

Simplified equation: provided that Q is not very low, the term cosh is near 1 and the term under sinh is near 0, so an approximate result is:

$$I_0 \approx V_M \sqrt{\frac{C}{L}} \cdot e^{-\frac{n\pi}{4Q}} \cos \frac{t_q}{2\sqrt{LC}}$$

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