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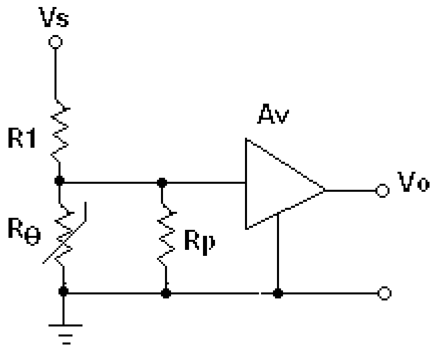
MODIFICATION OF NTC RESISTOR RESOLUTION BY MEANS OF A PARALLEL RESISTOR



The circuit shows a typical NTC resistor (thermistor) application. The resistance of a thermistor is given by the exponential equation:

$$R\theta = \alpha \cdot e^{\frac{\beta}{T}} \qquad (1)$$

where α and β are constants and T is the absolute temperature. The slope of this equation is its derivative:

$$\frac{\partial R\theta}{\partial T} = -\alpha \cdot \beta \cdot \frac{1}{T^2} \cdot e^{\frac{\beta}{T}} \qquad (2)$$

This slope decreases very fast when temperature increases, which means that resolution is much higher at low than at high temperatures. To increase resolution in the high temperature range, a parallel resistor Rp is added. The effect of this resistor will be more important when the thermistor resistance is higher, so intuitively it can be assumed that it will tend to equalize resolution in all the temperature range.

We will try to deduct an accurate value of this parallel resistor in order to optimise resolution in a certain range.

The output voltage is related to the supply voltage by the equation:

$$\frac{Vo}{Vs} = \frac{Av}{1 + \frac{R1}{R\theta} + \frac{R1}{Rp}} \qquad (3)$$

We compensate the change in output voltage caused by changes in Rp with an amplifier with gain:

$$Av = 1 + \frac{R1}{R\theta_{Max}} + \frac{R1}{Rp} \qquad (4)$$

so that when Rθ reaches it maximum value, Vo = Vs, the gain of the circuit becomes:

$$G = \frac{Vo}{Vs} = \frac{1+\frac{R1}{R\theta_{Max}}+\frac{R1}{Rp}}{1+\frac{R1}{R\theta}+\frac{R1}{Rp}} \quad (5)$$

The slope of output voltage function relative to the thermistor resistance is found calculating the derivative of the function (5):

$$S = \frac{\partial G}{\partial R\theta} = \frac{-\left(1+\frac{R1}{R\theta_{Max}}+\frac{R1}{Rp}\right)\cdot\frac{-R1}{R\theta^2}}{\left(1+\frac{R1}{R\theta}+\frac{R1}{Rp}\right)^2} = \frac{\frac{1}{R1}+\frac{1}{R\theta_{Max}}+\frac{1}{Rp}}{\left(1+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)^2} \quad (6)$$

We want to find the value of Rp which makes this slope maximum for a certain value of Rθ. We compute a new derivative of (6), but the variable is now Rp:

$$\frac{\partial S}{\partial Rp} = \frac{\left(1+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)^2\cdot\frac{-1}{Rp^2}-\left(\frac{1}{R1}+\frac{1}{R\theta_{Max}}+\frac{1}{Rp}\right)\cdot 2\cdot\left(1+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)\cdot\frac{-R\theta}{Rp^2}}{\left(1+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)^4} =$$

$$= \frac{2\cdot\left(\frac{R\theta}{R\theta_{Max}}+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)-\left(1+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)}{Rp\cdot\left(1+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)^2} = \frac{\frac{2\cdot R\theta}{R\theta_{Max}}+\frac{R\theta}{R1}+\frac{R\theta}{Rp}-1}{Rp\cdot\left(1+\frac{R\theta}{R1}+\frac{R\theta}{Rp}\right)^2} \quad (7)$$

To found the necessary condition for a maximum we equate (5) to 0 and obtain:

$$Rp = \frac{1}{\frac{1}{R\theta}-\frac{2}{R\theta_{Max}}-\frac{1}{R1}} \quad (8)$$

We omit the tedious task of showing that including this value in (7) and differentiating again, the result is negative, so the function is a maximum.

A physical interpretation of (8) is that the optimum value of Rp is the one that makes

$R\theta = Rp \,||\, R1 \,||\, R\theta_{Max} / 2$

Where the symbol || means paralleling resistors, so Rp has the value such that the parallel combination of Rp, R1 and $R\theta_{Max} / 2$ have the same value as Rθ.

The equation shows that a true optimisation in a point is only possible for certain values of Rθ and R1. For example in the particular case of the thermistor and Rp fed by a current source, the term 1/R1 becomes 0. Then a maximum exists only if $R\theta < R\theta_{Max} / 2$.