Losses in PWM inverters using IGBTs

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Abstract: The aim of this paper is to calculate the forward and switching losses of a PWM inverter employing IGBTs using a relatively simple method from manufacturers' catalogue parameters, that is, finding a reasonable compromise between accuracy and complexity. The system will be able to calculate losses for different modulation methods, provided the current output is sinusoidal. Part of the results can be used for other devices, such as bipolar transistors or MOSFETs.

List of symbols

E_{on} = turn on energy loss
E_{off} = turn off energy loss
E_d = energy loss in diode
E_i = energy loss in IGBT
F_s = switching frequency
I_C = collector current
I_{CN} = rated I_C
I_{CM} = maximum collector current
i_C = instantaneous collector current
I_{rr} = recovery current
I_{rrN} = rated recovery current
M = modulation depth
P_i = power loss in IGBT
P_d = power loss in diode
P_{on} = turn on power loss
P_{off} = turn off power loss
P_{rr} = recovery power loss
Q_{rr} = recovery charge
Q_{rrN} = rated recovery charge
t = period
t_f = fall time
t_{f*} = rated fall time
t_r = rise time
t_{rN} = rated rise time
t_{rr} = recovery time
t_{rrN} = rated recovery time
V_{cc} = DC bus voltage
V_{CE} = collector-to-emitter voltage
V_{CEN} = rated V_{CE}
V_{CE0} = threshold V_{CE}
V_F = diode forward voltage
V_{FN} = rated V_F
V_{F0} = diode threshold voltage
α = angle of current function
δ = duty cycle
θ = current lagging angle to voltage
τ = carrier period

1 Introduction

To calculate semiconductor losses in an inverter the energy loss in every pulse during a cycle needs to be calculated and then added. In a three-phase inverter the current in one phase depends on the state of the switches of the other phases and on the motor characteristics so the waveform is very complex. The calculation of semiconductor losses is feasible using a computer program which produces the pulse pattern and has a motor model. To do this by using equations derived from analytical methods, would be a quite impossible task, mainly because of the complex modulation strategies design of the third harmonic injection.

This approach has been tried using the method given in Reference 1, which starts with a pulse by pulse calculation (for only one kind of modulation strategy), but with the contradictory statement that the current is sinusoidal (no ripple), which only makes sense if the number of pulses tends to infinity. (Infinity here means 'as large a number as necessary to obtain a result that is below the allowed error'.) To simplify the equations, at the end of the calculations it is assumed that the number of pulses is high. As will be seen, if this is assumed from the start, calculation is much easier and it is feasible to obtain equations for other modulation strategies.

Reference 2, on bipolar transistors, assumes that the collector-to-emitter voltage is constant, which cannot be assumed with IGBTs. It seems to use a wrong switching pattern (assuming that current does not transfer from the fly wheeling diode to the transistor).

High switching frequencies used today (3 kHz minimum), give harmonic content of less than 5%. As forward losses depend on current, even if the device is only resistive (MOSFET), this cannot introduce an error greater than about 10%. Because an IGBT is not purely resistive, the error will be lower. In a usual IGBT inverter, switching at around 6 kHz, switching losses account for about 50% of the overall losses, so the total error in forward losses will be halved. An error of 5% for a temperature drop of 40°C would give an error of only 2°C in heat sink temperature in the junction. The error introduced, assuming a sinusoidal current, will be low for practical applications. It will always be much lower than the device parameter variation.

2 Forward losses

The typical voltage/current graph V_{CE}/I_{CE} (Fig. 1) is approximated by the following linear equation (this implies a threshold voltage plus a resistive voltage drop, the same assumption is made in Reference 1):

$$V_{CE} = \frac{V_{CEN} - V_{CE0}}{I_{CN}} I_C + V_{CE0} \tag{1}$$

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where I_{CN} and V_{CEN} are the rated (manufacturers catalogue) current and the collector-to-emitter voltage at the rated current.

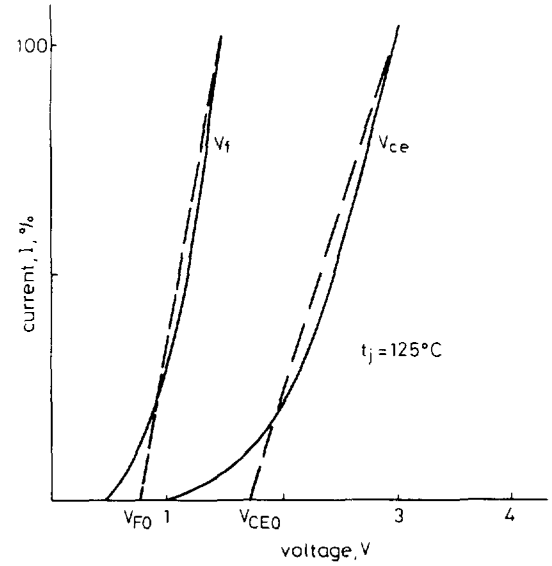

Fig. 1 *IGBT and diode forward voltage*

Note that all data should be taken at $T_j = 125°C$, the errors in data with the usual $T_j \geqslant 100°$ will be much lower than those obtained at 25°C.)

The value of the threshold voltage V_{CO} depends on the type of IGBT used and the rated voltage. For second generation 1200 V types, a convenient value for V_{CO} is about 1 V (at 125°C).

The diode forward voltage follows an exponential law. In the working range, we may simplify the equation, approximating it to a linear law with the origin at V_{FO} (see Fig. 1). This threshold voltage may be taken as 0.7 V

$$V_F = \frac{V_{FN} - V_{FO}}{I_{CN}} I_C + V_{FO} \qquad (2)$$

where V_{FN} is the diode voltage drop at rated current.

If $F(\alpha)$ is the modulating function $(-1 \leqslant F(\alpha) \leqslant 1)$, θ the angle between current and voltage and M the modulation index $(0 \leqslant M \leqslant 1)$, then the duty cycle δ of the voltage pulses, referred to the current angle α (Fig. 2) is

$$\delta = \tfrac{1}{2}[1 + MF(\alpha + \theta)] \qquad (3)$$

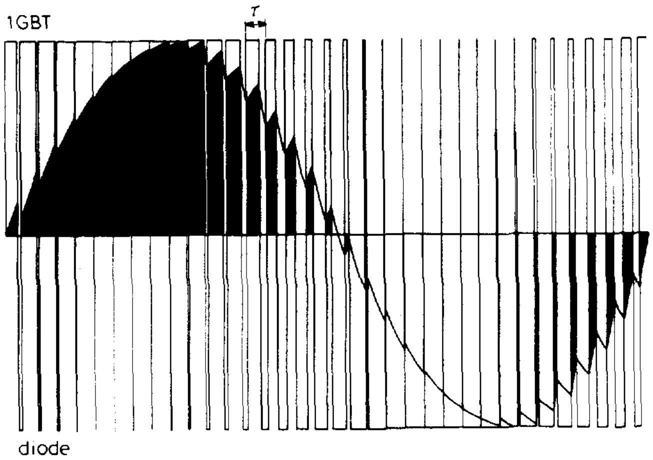

Fig. 2 *IGBT and diode currents*

As shown in Fig. 2, from $\alpha = 0$ to $\alpha = \pi$, the current circulates through the main switch during a time $\delta\tau$ of the carrier period τ, and through the opposite diode during the time $(1 - \delta)\tau$. During the other half cycle ($\alpha = \pi$–2π), the IGBT is off, and only its parallel diode works.

Consequently, we need to calculate only the overall losses during half a period, in order to obtain the total losses of a switch (IGBT + diode). The energy loss of an IGBT during a carrier period is

$$E_i = v_{CE} i_C \delta\tau$$
$$= \left(\frac{V_{CEN} - V_{CO}}{I_{CN}} i_C + V_{CO}\right) i_C \tfrac{1}{2}[1 + MF(\alpha + \theta)]\tau \qquad (4)$$

As we assume the current to be sinusoidal,

$$i_C = I_{CM} \sin \alpha$$

and

$$E_i = \left(\frac{V_{CEN} - V_{CO}}{I_{CN}} I_{CM} \sin \alpha + V_{CO}\right) I_{CM}$$
$$\times \sin \alpha \tfrac{1}{2}[1 + MF(\alpha + \theta)]\tau \qquad (5)$$

or

$$E_i = \tfrac{1}{2} I_{CM} \left(\frac{V_{CEN} - V_{CO}}{I_{CN}} I_{CM} \sin^2 \alpha + V_{CO} \sin \alpha\right)$$
$$\times [1 + MF(\alpha + \theta)]\tau \qquad (6)$$

Steady state power losses will not change if the number of pulses increases, because energy per pulse decreases (because τ decreases) in the same proportion as the number of pulses increases, so the total energy will remain the same.

This means that eqn. 6 may be converted to a differential equation. The average energy in a cycle will be the integral of the differential energy during half a period. The power will be the average energy divided by the full period

$$dt = \frac{d\alpha}{2\pi} T \quad dE_i = \frac{T}{2\pi} Ei \, d\alpha \quad P_i = \frac{1}{T} \int dE_i \qquad (7)$$

giving

$$P_i = \frac{1}{4\pi} I_{CM} \int_0^{\pi} \left(\frac{V_{CEN} - V_{CO}}{I_{CN}} I_{CM} \sin^2 \alpha + V_{CO} \sin \alpha\right)$$
$$\times [1 + MF(\alpha + \theta)] \, d\alpha \qquad (8)$$

Similarly, the diode energy loss is

$$E_d = v_F i_C (1 - \delta)\tau$$
$$= v_F i_C \tfrac{1}{2}[1 - MF(\alpha + \theta)]\tau \qquad (9)$$

which, as calculated for the IGBT, gives

$$E_d = \tfrac{1}{2} I_{CM} \left(\frac{V_{CEN} - V_{FO}}{I_{CN}} I_{CM} \sin^2 \alpha + V_{FO} \sin \alpha\right)$$
$$\times [1 - MF(\alpha + \theta)]\tau \qquad (10)$$

And the power loss is

$$P_d = \frac{1}{4\pi} I_{CM} \int_0^{\pi} \left(\frac{V_{CEN} - V_{FO}}{I_{CN}} I_{CM} \sin^2 \alpha + V_{FO} \sin \alpha\right)$$
$$\times [1 - MF(\alpha + \theta)] \, d\alpha \qquad (11)$$

In the case of sine modulation

$$F(\alpha + \theta) = \sin(\alpha + \theta) \qquad (12)$$

And the integration of eqns. 8 and 10 gives

$$P_i = \left(\frac{1}{8} + \frac{M}{3\pi}\right)\frac{V_{CEN} - V_{C0}}{I_{CN}} I_{CM}^2 \qquad \text{see note 1}$$

$$+ \left(\frac{1}{2\pi} + \frac{M}{8}\cos\theta\right) V_{C0} I_{CM}$$

$$P_d = \left(\frac{1}{8} - \frac{M}{3\pi}\right)\frac{V_{FN} - V_{F0}}{I_{CN}} I_{CM}^2 \qquad \text{see note 1}$$

$$+ \left(\frac{1}{2\pi} - \frac{M}{8}\cos\theta\right) V_{F0} I_{CM} \tag{13}$$

Note that these results are the same as those given in Reference 1. If, in eqn. 13 we made $V_{C0} = 0$ and $V_{F0} = 0$ the equations given in the introductory part of Reference 3 are obtained (although there seems to be a sign error in the diode losses).

For sine modulation with the third harmonic

$$F(\alpha + \theta) = \frac{2}{\sqrt{(3)}}\left[\sin(\alpha + \theta) + \tfrac{1}{6}\sin(3(\alpha + \theta))\right] \tag{14}$$

Which, when integrated, gives

$$P_i = \left[\frac{1}{8} + \frac{2\sqrt{(3)}}{9\pi} M\cos\theta - \frac{\sqrt{(3)}}{45\pi} M\cos 3\theta\right]$$

$$\times \frac{V_{CEN} - V_{C0}}{I_{CN}} I_{CM}^2$$

$$+ \left(\frac{1}{2\pi} + \frac{\sqrt{(3)}}{12} M\cos\theta\right) V_{C0} I_{CM}$$

$$P_d = \left[\frac{1}{8} - \frac{2\sqrt{(3)}}{9\pi} M\cos\theta + \frac{\sqrt{(3)}}{45\pi} M\cos 3\theta\right]$$

$$\times \frac{V_{CEN} - V_{C0}}{I_{CN}} I_{CM}^2$$

$$+ \left(\frac{1}{2\pi} - \frac{\sqrt{(3)}}{12} M\cos\theta\right) V_{C0} I_{CM} \tag{15}$$

These last equations are important because they can be used for other modulation systems which also give 100% output voltage, such as bus clamping and vector modulation. Results, using numerical integration, showed insignificant differences (less than 0.2%) when these equations were used. This is not surprising, because the voltage average waveforms are very similar. From a theoretical point of view, it is interesting to note that if the modulation depth is divided by the factor $2/\sqrt{(3)}$ (the overmodulation ratio of these strategies), the results are very close to those obtained from eqns. 13.

MOSFET inverter: Previous equations can be used for MOSFETs with

$$V_{C0} = 0 \quad V_{CEN} = I_{CMN} R_{on}$$

and for other kinds of power switch, with the appropriate value of V_{C0}.

3 Switching losses

3.1 Turn-on losses

If the IGBT were an ideal switch, with no delay times, turn-on di/dt would be V_{cc}/L, where L is the self-inductance of the circuit. In practical circuits with low inductance and reasonable gate resistor values, rise time is determined basically by the gate resistor. Manufacturers' catalogues show the rise time for different gate resistor values.

Fig. 3 shows typical switching waveforms. di/dt during turn-on is fairly constant. This means that rise time can

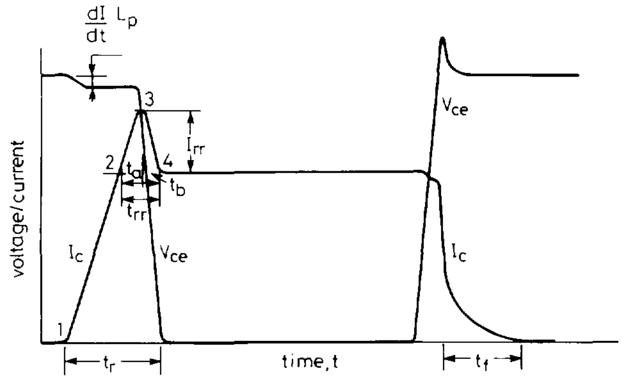

Fig. 3 *Typical switching waveforms*

be considered to be proportional to the switching current. If t_{rN} is the rated rise time at rated current

$$E_{on} = \int_0^{t_r} V_{cc}\frac{i_C}{t_r} t\, dt = \tfrac{1}{2} V_{cc}\, i_C\, t_r$$

$$= \tfrac{1}{2} V_{cc}\, i_C\, t_{rN} \frac{i_C}{I_{CN}}$$

$$E_{on} = \tfrac{1}{2} V_{cc}\, t_{rN} \frac{I_{CM}^2 \sin^2\alpha}{I_{CN}} \tag{16}$$

The average energy loss in the full period is

$$E_{on(av)} = \frac{1}{2\pi}\int_0^{\pi} E_{on}\, d\alpha$$

$$= \frac{1}{4\pi} V_{cc}\, t_{rN} \frac{I_{CM}^2}{I_{CN}} \int_0^{\pi} \sin^2\alpha\, d\alpha$$

$$E_{on(av)} = \tfrac{1}{8} V_{cc}\, t_{rN} \frac{I_{CM}^2}{I_{CN}} \tag{17}$$

so the power, at switching frequency F_s is

$$P_{on} = \tfrac{1}{8} V_{cc}\, t_{rN} \frac{I_{CM}^2}{I_{CN}} F_s \tag{18}$$

3.2 Turn-off losses

From manufacturers' catalogues and experimental data [4–7], it is clear that fall time (t_f) does not change much with current. This fact may be explained as follows.

From Fig. 3, it can be seen that the initial rate of decay is very fast. However, when the current reaches a certain limit, it decreases logarithmically (tail current). Therefore the predominant time, unless the current is very low, is due to the current tail characteristic.

Fall time increases greatly with temperature, so data at 25°C are absolutely meaningless. The change of fall time with current depends on the proportion of tail current time over the initial fall time. Although data from manufacturers differ, as a general rule, at 125°C, fall time increases by about 40% when current changes from 20 to 100% of its rated value. This can be approximated with the linear function

$$t_f \approx \left(\frac{2}{3} + \frac{1}{3}\frac{i_C}{I_{CN}}\right) t_{fN} \tag{19}$$

Note 1: Errata. Instead of M, must say M cosθ

Table 1: Comparison of experimental and measured losses

Parameter					
I_{CEN}, A	15	15	15	15	75
I_{out}, RMS, A	3.9	5.2	5	2.85	24
V_{CEN}, V	2.5	2.5	2.5	2.5	2.5
V_{FN}	1.8	1.8	1.8	1.8	2.2
V_{cc}, V	580	580	540	580	580
F_s, Hz	6000	5700	10 800	5400	5700
cos θ	0.8	0.8	0.8	0.8	0.85
M, %	100	90	90	90	95
t_r, ns	200	200	200	200	200
t_f, ns	200	200	200	200	300
Q_{rr}, nC	200	200	200	200	1100
t_{rr}, ns	200	200	200	200	200
Ambient temperature, °C	40	42	26	42	42
Heat sink, °C/W	0.54	0.54	0.54	0.54	0.22
Extra losses, W	11.7	12.9	12.2	9.2	45
Heat sink, °C	62	65	54.6	55.3	90
Calculated values					
Heat sink, °C	60.3	68.4	57.2	56.1	90.3
Losses in W per switch					
IGBT on losses	2.2	3.1	2.9	1.4	14.5
Diode on losses	0.2	0.4	0.4	0.2	1.6
Turn-on and recovery losses	1.5	1.9	3.2	0.9	8.7
Turn-off losses	0.5	0.6	1.0	0.3	4.2

And, as before

$$E_{off} = \tfrac{1}{2} V_{cc}\, i_C\, t_{fN}$$

$$E_{off} = V_{cc}\, I_{CM}\left(\tfrac{1}{3}\sin\alpha + \tfrac{1}{6}\sin^2\alpha\,\frac{I_{CM}}{I_{CN}}\right)t_{fN}$$

$$E_{off(av)} = \frac{1}{2\pi}\int_0^{\pi} E_{off}\, d\alpha \qquad (20)$$

The power loss is

$$P_{off} = V_{cc}\, I_{CM}\, t_{fN}\, F_s\left(\frac{1}{3\pi} + \frac{1}{24}\frac{I_{CM}}{I_{CN}}\right) \qquad (21)$$

If the fall time had been considered to be constant the factor 0.93 would have been 1.

During the fast falling period, some overvoltage occurs. This could be taken into account in the calculation. However, the *di/dt* is only very high for a short time, so the overvoltage pulse is very short and the increase in losses is relatively very small.

4 Recovery losses

Eqn. 18 gives us the losses for the switching interval 1 to 2 (Fig. 3). After time 2, the current in the IGBT increases beyond the load current, owing to the stored charges in the opposite diode. At time 3, a depletion region is formed, the diode begins to support the voltage, stored charges disappear by recombination, and the collector voltage begins to fall. At time 4, recovery current is zero (in fact 10% of I_{rr}).

From time 2 to time 3, V_{cc} is supported by the IGBT. Almost all losses are generated in it. Current follows in a near triangular shape, so with I_{rr} being the peak recovery current, the IGBT current may be calculated as a linear function of i_C

$$i = I_{rr}\frac{t}{t_a} + i_C \qquad (22)$$

The recovery time (t_{rr}) and I_{rr} change with temperature (increasing on average 70% from 25°C to 125°C; this seems to be the disadvantage of using a minority charge killer). I_{rr} increases in the same proportion or may even double, between 25°C and 125°C. As Q_{rr} is the product of I_{rr} and t_{rr} (divided by 2), it means that Q_{rr} increases at least three times between 25°C and 125°C. This explains why some manufactures give useless data at 25°C.

Too often, recovery data are specified at a *di/dt* which is much lower than the normal working value. As a manufacturers' catalogue example, the typical rise time of an 100 A IGBT is specified as 300 ns. This gives a typical *di/dt* of 330 A/μs. However, recovery time is specified at 100 A/μs (and at 25°C, of course).

The relationship between t_{rr} and the current is taken from manufacturer data (see References 4–7 and catalogues). Averaging different measurements at 125°C, the following linear relationship is approximated

$$t_{rr} \approx \left(0.8 + 0.2\frac{i_C}{I_{CN}}\right)t_{rrN} \qquad (23)$$

And with these value in eqn. 22, integrating up to t_a we obtain

$$E_{rra} = V_{cc}\, t_a\left(0.35 I_{rrN} + 0.15\frac{i_C}{I_{CN}} I_{rrN} + i_C\right) \qquad (24)$$

From time 3 to time 4, losses are generated in the diode and in the IGBT. Voltage reaches zero at a point which corresponds to about the full recovery of the diode.

The instantaneous power loss in the IGBT is $V_{CE}\, i_C$, and in the diode, $(V_{cc} - V_{CE})i_C$. The total power loss is $V_{CE}\, i_C + (V_{cc} - V_{CE})i_C = V_{cc}\, i_C$, exactly the same as that during time t_a. So, calculation is simplified if diode and transistor losses do not need to be calculated separately. The equation used will be eqn. 23, substituting t_b for t_a.

The total recovery energy is

$$E_{rr} = V_{cc}\, t_{rr}\left(0.35 I_{rrN} + 0.15\frac{i_C}{I_{CN}} I_{rrN} + i_C\right)$$

$$E_{rr} = V_{cc}\, t_{rrN}\left(0.8 + 0.2\frac{i_C}{I_{CN}}\right) \times \left(0.35 I_{rrN} + 0.15\frac{i_C}{I_{CN}} I_{rrN} + i_C\right) \qquad (25)$$

And the total power loss is

$$P_{rr} = \frac{F_s}{2\pi} \int_0^{\pi} E_{rr}\, d\alpha \qquad (26)$$

Taking into account that $i_C = I_{CM} \sin \alpha$

$$P_{rr} = F_s V_{cc} \left[\left(0.28 + \frac{0.38}{\pi} \frac{I_{CM}}{I_{CN}} + 0.015 \left(\frac{I_{CM}}{I_{CN}} \right)^2 \right) \times Q_{rrN} + \left(\frac{0.8}{\pi} + 0.05 \frac{I_{CM}}{I_{CN}} \right) I_{CM}\, t_{rrN} \right] \qquad (27)$$

Two interesting cases are

$$I_{CM} = I_{CN}$$

$$P_{rr} = F_s V_{cc}(0.416 Q_{rrN} + 0.305 I_{CM}\, t_{rrN})$$

$$I_{CM} = \tfrac{2}{3} I_{CN}$$

$$P_{rr} = F_s V_{cc}(0.367 Q_{rrN} + 0.288 I_{CM}\, t_{rrN}) \qquad (28)$$

The second case corresponds to a load current allowing 150% overload. If recovery time and current had been considered to be constant, the coefficients of the equation would have been 1/2 and $1/\pi$.

Note that in the case of 'bus clamping' modulation, the switching frequency has to be taken as 2/3 of its real value, as every IGBT is inactive for 1/3 of the cycle.

5 Comparison with measured losses

Measurements were made in a new range of three-phase IGBT inverters, with sine modulation with the third harmonic injection. Heat sink thermal resistance was measured using resistors giving the expected power. IGBT parameters were checked using an oscilloscope and found to be close to typical values. The IGBTs used were Mitsubishi, E range (1200 V); $V_{CE0} = 1$ V, $V_{F0} = 0.7$ V. All IGBTs were mounted in the same heat sink, with the rectifier. 'Extra losses' include rectifier losses and losses of a MOSFET attached to the same heat sink. The experimental and calculated results are given in Table 1.

6 Conclusion

The results for on state losses can be used for any power switch, by just modifying the value of the threshold voltage V_{C0}, and are useful for almost any modulation strategy.

The equations for turn-on losses may be applied to most semiconductors. However the equations for turn-off and recovery losses have to be regarded with more circumspection, as they depend mainly on the physics of a particular device (i.e. method for getting rid of the stored charges) and can only be applied to IGBTs of the second generation. However, the procedure shown can be easily followed to suit other IGBTs of future generations, to adapt them better to a particular brand or to use them for other devices.

From the experiments and computer simulation, it can be deduced that conducting and switching losses can be calculated quite accurately, provided that the device parameters are well known. As a practical rule, a result with an average error of 10% or less should be expected if parameters are correct. The worst case that was found had 15% error. No correlation with the size of error and the power factor or modulation index was found.

At the same modulation depth, sine modulation gives slightly less IGBT conducting losses (about 6%) and more diode losses than other methods, but this is because it only reaches 86.6% of the full voltage. Third harmonic injection or vector modulation at 86.6% of modulation depth, give almost the same results.

Third harmonic injection, vector modulation and bus clamping all give the same forward losses (less than 0.5% difference was found in the computer simulation), so eqns. 15 can be used for all these methods. Eqns. 15 can be considered to be general equations if, for sine modulation, modulation depth is multiplied by the factor $\sqrt{(3/2)}$. As expected, bus clamping gives less switching losses (2/3) than the other methods.

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